

Final Exam Study Guide

1. Given the function, $f(x) = -2x^2 + x - 1$, find
 - a. $f(0)$
 - b. $f(1)$
 - c. $-f(x)$
 - d. $f(x+h)$
2. Find the domain of each function
 - a. $f(x) = x^2 + 2$
 - b. $f(x) = \frac{x}{\sqrt{x-4}}$
 - c. $f(x) = \sqrt{\frac{2}{x-1}}$
3. For the given functions, f , find the difference quotient; that is, find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$.
 - a. $f(x) = 4x^2 + 5x - 7$
 - b. $f(x) = \frac{1}{x+3}$
4. If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately $H(x) = 20 + 13x^2$
 - a. What is the height of the rock when $x = 1$ seconds? $x = 1.1$ seconds? $x = 1.2$ seconds?
 - b. When is the height of the rocks 15 meters? When is it 10 meters? When is it 5 meters?
 - c. When does the rock strike the ground?
5. $f(x) = -3x^2 + 5x$
 - a. Is the point $(-1, 2)$ on the graph of f ?
 - b. If $x = -2$, what is $f(x)$? What point is on the graph of f ?
 - c. If $f(x) = -2$, what is x ? What point(s) are on the graph of f ?
 - d. What is the domain of f ?
 - e. List the x -intercept, if any, of the graph of f ?
 - f. List the y -intercept, if there is one, of the graph of f ?
6. Determine algebraically whether each function is even, odd, or neither.
 - a. $f(x) = 2x^4 - x^2$
 - b. $h(x) = \frac{x}{x^2 - 1}$
 - c. $h(x) = \frac{-x^3}{3x^2 - 9}$

7. Find the average rate of change of $h(x) = x^2 - 2x + 3$
- From -1 to 1
 - From 0 to 2
 - From 2 to 5
8. If $f(x) = \begin{cases} -3x & , x < -1 \\ 0 & , x = -1 \\ 2x^2 + 1 & , x > -1 \end{cases}$
- Find $f(-2)$
 - Find $f(-1)$
 - Find $f(0)$
9. If $f(x) = \begin{cases} x^3 & , -2 \leq x < 1 \\ 3x + 2 & , 1 \leq x \leq 4 \end{cases}$
- Find $f(-1)$
 - Find $f(0)$
 - Find $f(1)$
 - Find $f(3)$
10. Find the function that is finally graphed after the following transformations are applied to the graph of $f(x) = \sqrt{x}$
- Reflect about the x -axis
 - Shift right 3 units
 - Shift down 2 units
11. For the following problems, graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function, and show all stages.
- $f(x) = (x + 2)^3 - 3$
 - $f(x) = -\sqrt{x}$
 - $g(x) = -\frac{1}{x}$
 - $g(x) = \sqrt{x - 2} + 1$
 - $f(x) = -(x + 1)^3 - 1$
12. A rectangle is inscribed in a semicircle of radius 2. See the figure. Let $P = (x, y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle.
- Express the area A of the rectangle as a function of x .
 - Express the perimeter p of the rectangle as a function of x .
13. A wire of length x is bent into the shape of a square.
- Express the perimeter of the square as a function of x .
 - Express the area of the square as a function of x .
14. For the following functions, (a) graph each quadratic function by determining whether its graph opens up or down and by finding its vertex, axis of

- symmetry, y -intercept, and x -intercept, if any. (b) Determine the domain and the range of the function. (c) Determine where the function is increasing and where it is decreasing.
- $f(x) = x^2 - 4x$
 - $f(x) = 3x^2 + 18x$
 - $f(x) = x^2 + 6x + 9$
 - $f(x) = 4x^2 - 2x + 1$
 - $f(x) = 3x^2 + 6x + 2$
15. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{3}x + 100$, $0 \leq x \leq 300$.
- Express the revenue R as a function of x .
 - What is the revenue if 100 units are sold?
 - What quantity x maximizes revenue? What is the maximum revenue?
 - What price should the company charge to maximize revenue?
16. Beth has 3000 feet of fencing available to enclose a rectangular field.
- Express the area A of the rectangle as a function of x , where x is the length of the rectangle.
 - For what value of x is the area largest?
 - What is the maximum area?
17. A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?
18. Form a polynomial whose real zeros and degree are given
- Zeros: -3, -1, 2, 5; degree 4
 - Zeros: -2, multiplicity 2; 4, multiplicity 1; degree 3
19. Find parts a - e for $f(x) = 2(x - 3)(x + 4)^3$
- List each real zero and its multiplicity
 - Determine whether the graph crosses or touches the x -axis at each x -intercept
 - Determine the behavior of the graph near each x -intercept
 - Determine the maximum number of turning points on the graph
 - Determine the end behavior; that is, find the power function that the graph of f resembles for large values of x .
20. Find parts a - e for $f(x) = -x^2(x^2 - 4)(x - 5)$
- List each real zero and its multiplicity
 - Determine whether the graph crosses or touches the x -axis at each x -intercept
 - Determine the behavior of the graph near each x -intercept
 - Determine the maximum number of turning points on the graph

e) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of x .

21. Find the vertical, horizontal, and oblique asymptotes, if any, of each rational function

a) $f(x) = \frac{3x+5}{x-6}$

b) $f(x) = \frac{-x^2+1}{x^2-5x+6}$

c) $f(x) = \frac{4x^5}{x^3-1}$

d) $f(x) = \frac{-2x^2+1}{2x^3+4x^2}$

e) $f(x) = \frac{x-1}{x-x^3}$

22. Graph the following rational functions (7 - step process)

a) $f(x) = \frac{x^2+x-12}{x^2-4}$

b) $f(x) = \frac{x^2-x-12}{x+5}$

c) $f(x) = \frac{x^2+5x+6}{x+3}$

23. Solve each inequality

a) $x^3 - 2x^2 - 3x > 0$

b) $x^4 < 9x^2$

c) $\frac{(x-3)(x+2)}{x-1} \leq 0$

d) $\frac{x+2}{x-4} \geq 1$

e) $\frac{1}{x+2} > \frac{3}{x+1}$

24. List the **potential** zeros then use Synthetic division to find the **actual** zeros

a) $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$

b) $f(x) = 2x^3 + 3x^2 + 2x + 3$

25. Form a polynomial $f(x)$ having the given degree and zeros

a) Zeros: $3+2i$; 4 , multiplicity 2

b) Zeros: i , $1+3i$

26. For parts $a - d$, use $f(x) = |x - 2|$; $g(x) = \frac{3}{x^2+2}$

a) $(f(g(4))) =$

b) $(g(f(2))) =$

c) $(f(f(1))) =$

d) $(g(g(0))) =$

27. Find the domain of the composite function $(f(g(x)))$

a) $f(x) = \frac{1}{x+3}$; $g(x) = \frac{2}{x}$

b) $f(x) = x^2 + 4$; $g(x) = \sqrt{x-2}$

28. For parts $a - d$, use $f(x) = \sqrt{x-2}$; $g(x) = 1 - 2x$. Also find the domain of each function

a) $(f(g(x))) =$

b) $(g(f(x))) =$

c) $(f(f(x))) =$

d) $(g(g(x))) =$

29. Verify that the functions $f(x)$ and $g(x)$ are inverses of each other by showing that

$(f(g(x))) = x$ and $(g(f(x))) = x$

a) $f(x) = 2x + 6$; $g(x) = \frac{1}{2}x - 3$

b) $f(x) = (x - 2)^2$; $g(x) = \sqrt{x} + 2$

30. Each function is one-to-one. Find its inverse and check your answer. State the domain of f and find its range using f^{-1}

a) $f(x) = \frac{4}{2-x}$

b) $f(x) = \frac{2x-3}{x+4}$

31. Use Transformations to graph $f(x)$

a) $f(x) = 2 + 4^{x-1}$

b) $f(x) = 3^{-x} - 2$

c) $f(x) = 5 - e^{-x}$

32. Solve each equation

a) $3^{-x} = 81$

b) $5^{x+3} = \frac{1}{5}$

c) $9^{-x+15} = 27^x$

d) $9^{2x} \cdot 27^{x^2} = 3^{-1}$

e) $(e^4)^x \cdot e^{x^2} = e^{12}$

33. Change each exponential expression to an equivalent expression involving a logarithm

a) $9 = 3^2$

b) $16 = 4^2$

c) $2^x = 7.2$

d) $3^x = 4.6$

34. Change each logarithmic expression to an equivalent expression involving an exponent

a) $\log_2 8 = 3$

b) $\log_3 \frac{1}{9} = -2$

c) $\log_2 3 = x$

d) $\log_2 6 = x$

35. Find the domain of each function

a) $f(x) = 8 + 5\ln(2x + 3)$

b) $f(x) = \ln\left(\frac{1}{x+1}\right)$

c) $f(x) = \log_5\left(\frac{x+1}{x}\right)$

36. Use the given function f to: $f(x) = -\ln(-x)$

a) Find the domain of f .

b) Graph f .

c) From the graph, determine the range and any asymptotes of f .

d) Find f^{-1} , the inverse of f .

e) Use f^{-1} to find the range of f .

f) Graph f^{-1} .

37. Write each expression as a sum and/or difference of logarithms. Express powers as factors.

a) $\log_5 \frac{\sqrt[3]{x^2+1}}{x^2-1}$

b) $\log \left[\frac{x(x+2)}{(x+3)^2} \right]$

c) $\ln \left[\frac{5x^2 \sqrt[3]{1-x}}{4(x+1)^2} \right]$

38. Write each expression as a single logarithm

a) $\log_2 \left(\frac{1}{x} \right) + \log_2 \left(\frac{1}{x^2} \right)$

b) $\log_4(x^2 - 1) - 5 \log_4(x + 1)$

c) $\log \frac{x^2+2x-3}{x^2-4} - \log \frac{x^2+7x+6}{x+2}$

d) $\frac{1}{3} \log(x^3 + 1) + \frac{1}{2} \log(x^2 + 1)$

39. Solve each logarithmic equation

a) $\log_8(x + 6) = 1 - \log_8(x + 4)$

b) $\log_{\frac{1}{3}}(x^2 + x) - \log_{\frac{1}{3}}(x^2 - x) = -1$

c) $\log_4(x^2 - 9) - \log_4(x + 3) = 3$

$$d) \log_a x + \log_a(x - 2) = \log_a(x + 4)$$

40. Solve each exponential equation

$$a) 3^{1-2x} = 4^x$$

$$b) \pi^{1-x} = e^x$$

$$c) 2^{2x} + 2^x - 12 = 0$$

$$d) 16^x + 4^{x+1} - 3 = 0$$

41. Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine 131.

- What is the decay rate of iodine 131?
- How much iodine 131 is left after 9 days?
- When will 70 grams of iodine 131 be left?
- What is the half-life of iodine 131?

42. Find the equation of the parabola described.

- Focus at $(0,-1)$; directrix the line $y = 1$
- Focus at $(2,4)$; directrix the line $x = -4$
- Focus at $(-4,4)$; directrix the line $y = -2$

43. Find the vertex, focus, and directrix of each parabola. Graph the equation.

- $x^2 + 8x = 4y - 8$
- $y^2 - 2y = 8x - 1$

44. The reflector of a flashlight is in the shape of a paraboloid of revolution. Its diameter is 4 inches and its depth is 1 inch. How far from the vertex should the light bulb be placed so that the rays will be reflected parallel to the axis?

45. A searchlight is shaped like a paraboloid of revolution. If the light source is located 2 feet from the base along the axis of symmetry and the opening is 5 feet across, how deep should the searchlight be?

46. Find the vertices and foci of each ellipse.

- $4x^2 + y^2 = 16$
- $4y^2 + 9x^2 = 36$

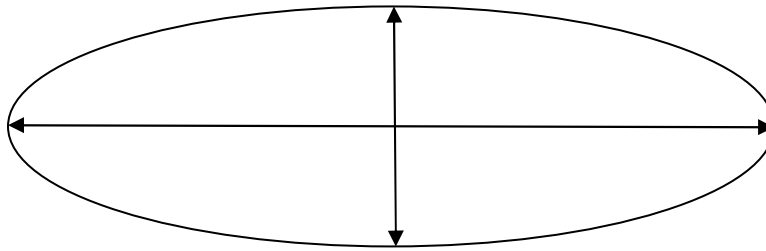
47. Find an equation for each ellipse

- a) Center at $(0,0)$; focus at $(-1,0)$; vertex at $(3,0)$
- b) Center at $(0,0)$; focus at $(0,1)$; vertex at $(0,-2)$

48. Find the center, vertices, and foci of each ellipse. Graph each equation.

- a) $4x^2 + 3y^2 + 8x - 6y = 5$
- b) $x^2 + 3^2 - 12y + 9 = 0$

49. A racetrack is in the shape of an ellipse, 100 feet long and 50 feet wide. What is the width 10 feet from a vertex?



50. Find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

- a) $4x^2 - y^2 = 16$
- b) $y^2 - 9x^2 = 9$
- c) $4y^2 - x^2 = 16$

51. Find an equation for the hyperbola described. Graph the equation.

- a) Center at $(1,4)$; Focus at $(-2,4)$; Vertex at $(0,4)$
- b) Focus at $(-4,0)$; vertices at $(-4,4)$ and $(-4,2)$

52. Find the Center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

- a) $y^2 - x^2 - 4y + 4x - 1 = 0$
- b) $2x^2 - y^2 + 4x + 4y - 4 = 0$